Grade 11/12 Math Circles

March 20, 2024

Primality Testing and Integer Factorization - Problem Set

1. Determine whether the following statements are true.
   - $16 \equiv 51 \pmod 5$
   - $21 \equiv 0 \pmod 7$
   - $4 \equiv 12 \pmod{16}$
   - $-4 \equiv 12 \pmod{16}$
2. Determine whether the following equalities are true:
   - $[-4] = [16] \pmod 5$
   - $[2] = [14] \pmod 7$.
3. Calculate 7^{200} % 48.
4. Calculate 11^{301} % 1332.
5. Calculate 3^k % 10 , for $0 \leq k \leq 12$. What do you notice?
6. Show that if $m \geq 1$ has any odd prime factor, that $2^m + 1$ is composite.
7. Show that if $m \geq 1$ is composite, then $2^m - 1$ is composite.
8. Verify that 561 is a Carmichael number.
9. Find the four roots of the polynomial $x^4 - 1$ mod 5.
10. Find a modulus m such that $x^2 + 1$ has two roots.
11. How many bases must we choose to theoretically have a 99% chance that m is prime?